Randomacci Sequences

Albert Natian Professor of Mathematics Los Angeles Valley College

Let $(Y_n)_{n=0}^{\infty}$ be a sequence of independent random variables such that for each n in $\{0, 1, 2, \cdots\}$: Y_n uniformly assumes a random number from the set $\{0, 1, 2, \cdots, n\}$.

Now define recursively a sequence $(X_n)_{n=0}^{\infty}$ of random variables as follows:

$$X_0 = 0, \quad X_1 = 1, \quad \forall n \in \{2, 3, \cdots\} : \quad X_n = X_{Y_{n-1}} + X_{Y_{n-2}}.$$

We call this sequence the *Randomacci* sequence. What sort of identities involving the Randomacci Sequence (and other sequences, say, Fibonacci, Lucas, etc.) can we discover? What are the distributions, expected values and variances of the following sampling of random variables?

$$\text{(i) } X_n \qquad \text{(ii)} X_{Y_n} \qquad \text{(iii) } Y_{X_n} \qquad \text{(iv) } X_{X_n} \qquad \text{(v) } \prod_{k=1}^{n} X_k \qquad \text{(vi) } \lim_{n\to\infty} X_n/X_{n-1}$$

$$\text{(vii) } F_{X_n} \qquad \text{(viii) } F_{Y_n} \qquad \text{(ix) } F_{X_n} \qquad \text{(x) } F_{Y_n} F_{X_n} \qquad \text{(xi) } \prod_{k=1}^{Y_n} X_k \qquad \text{(xii) } \sum_{k=1}^{X_n} Y_n.$$

Other important questions pertain the asymptotic behavior of expressions involving the Randomacci sequences (as well as other 'garden' variety sequences).